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The Innovative 24-Note Numerical System: A Computational Method and Model of the Relationships Between Notes, Elements, and Planets



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ABSTRACT

This research introduces a 24-note numerical framework and a computational method to examine the connections between music, mathematics, the theory of the four temperaments, and the constellations of the zodiac. Although previous studies have acknowledged these associations, they have lacked formal mathematical structures and systematic validation. The 24-note system is significant in two respects: first, it corroborates the existing 17-note system; and second, it establishes a vital theoretical foundation for microtones. By providing such a robust infrastructure, this research challenges the conventional Western musical framework—limited to whole tones and semitones—and paves the way for the practical application of microtones in fields such as music therapy, where their potential remains largely untapped. Despite the difference in the number of notes, the results demonstrate complete agreement between the 24-note and 17-note systems regarding the analysis of the scales associated with the seven primary *__Dastgahs__* of Iranian music. The study reveals that structural symmetry can identify symmetrical scales, but only when the sum of the "value numbers" (positions) in one tetrachord equals the sum in the other. Furthermore, the research introduces a highly innovative and valuable relational model linking musical notes to the four classical elements and the planets. The findings indicate that the note "A" (La) serves as the fundamental reference point, with the positions of other notes calibrated and determined in relation to it. Finally, the study presents the "Cyclic Boundary Theorem" (CBT); A novel mathematical concept that proposes two complementary computational approaches for the systematic numerical evaluation of segmented rotational systems.

1. Introduction:

Greek philosophers regarded music as a branch of the mathematical sciences. The Greek scholar Pythagoras believed that all things result from the combination of numbers and their ratios. In his writings, he viewed the combination of sounds to create a melody as a function of numerical equations [13]. In medieval culture, Boethius (6th century) revived the theory of a numerical balance between music, the cosmos, and the human body. This philosopher proposed that the connections between the universe and celestial planets could be conceptualized as a form of "***earthly music***" [16]. Iranian philosophers formulated twelve musical *__maqams__* (modes) corresponding to the twelve constellations of the Zodiac, which constituted their fundamental system [21]. Historical sources regarding the Zodiac primarily originate from various cuneiform

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texts—which were not exclusively astronomical in nature—dating back to the first millennium BCE [18].

Safi al-Din al-Urmawi (d. 1294), in his work titled ***Bahjat al-Ruh*** (The Joy of the Soul), highlighted the astro-musical connections between the twelve Iranian ***maqams*** and the constellations of the Zodiac. In a diagram, he linked each ***maqam*** to the views of earlier musicians and also specified the seasons and months appropriate for performing these Iranian musical modes. For instance, he has listed the melodies to be played in the spring—when the sun is in the signs of Aries (the Ram), Taurus (the Bull), or Gemini (the Twins)—and in the summer—when the sun is in the signs of Cancer (the Crab), Leo (the Lion), and Virgo (the Maiden)— and has similarly enumerated the other seasons and ***maqāms*** [24]. Based on natal astrology, the positions of the sun, moon, and planets at the moment of birth can be used to determine an individual's personality, temperament, and behavior [25].

In Persian, “temperament” means composition, and in traditional Iranian medicine, it refers to the unique quality that results from the mixing and interaction of the four elements of water, earth, air, and fire in a composite body [17]. These elements, which form the basis of all beings (*human and non-human*), are indivisible and cannot be divided into parts with distinct natures or temperaments. Their defining characteristics are as follows: fire (***hot and dry***), air (hot and moist), water (cold and moist), and earth (***cold and dry***) [7]. The followers of Hippocrates, in line with Empedocles, believed that everything, including humans, was composed of these four elements. They also attributed each element to a temperament of the body: earth to black bile, air to blood, fire to yellow bile, and water to phlegm. Health depends on the balance between these temperaments, while an imbalance causes illness [11]. Following the Greeks, Persian philosophers also examined the psychological and therapeutic effects of music through the framework of the four temperaments, which was supported by medical reasoning [20].

The ancients said that the bottom voice (soprano) represented yellow bile, the middle voice (alto) represented blood, the third part (tenor) represented phlegm, and the bass represented black bile [1]. Adapt your music to the temperament of your listeners: play tenor for capricious people (***rosy-faced***), soprano for choleric people (***yellow***), alto for melancholic listeners (dark, thin), and bass for phlegmatic listeners (pale, heavy), reflecting the ancient alignment of music with the four temperaments [19]. In autumn, when the Sun is in the house of Libra, Scorpio and Sagittarius, several melodies should be sung in accordance with their nature, i.e. dirtiness, coldness and dryness: such as Ashaq, Chahargah, Busalik, Bozorg, Kokush, Neyriz, Homayun, Ezal and Hosseini [12].

At present, there is incomplete information about the past steps, and the present-day radif, consisting of seven dastakhs and five avas, differs from the historical steps in terms of classification and corners. Therefore, the effect of current dastakhs and avas on the mood and spirit of the listeners needs psychological study, experimentation and interpretation [20]. With the rise of the Safavid dynasty (***1501 AD***) and its socio-political developments, maghami music gradually evolved. Although no document provides precise details of this process, scattered writings indicate that dastakhi music emerged in the early 13th century AH (1790 AD). [14]

The current Iranian radif music system consists of seven dastakhs (***mahor, shoor, homayun, se gah, chahargah, nava, rast panjgah***) and five avaz (***abu-ata, bayat-turk, afshari, dashti, isfahan***)[9]. A dastakh is a musical system consisting of several gushes, starting from a main “mother mode” that forms its basis. The performance is then modulated in a specific order through a series of modes, each section usually ending in a descent (cadence) that returns the melody to the mother mode [14]. The results of the GLM analysis show that temperament type is associated with neuroticism. This relationship is greater in people with a melancholic temperament and those who prefer the shoor dastakh [4].

By the time of al-Farabi (d. 950) [10], the two main intervals in the scale (whole step, half step) attributed to Pythagoras were accepted and practiced on the instruments of that time, and his contemporaries knew no other principle for intervals on instruments. [22]. Vilotto [23], the Western musicologist who pioneered Napoleon's campaign in Egypt, identified in his research two main approaches to Eastern scale systems: first, a theoretical 24-tone octave division using quarter notes and sometimes extending to 48 microtonal divisions. Second, a practical 18-tone system which divided each whole tone into three approximately equal parts (***approximately 1/3-tone intervals***). He conclusively demonstrated that this 18-tone division, despite the theoretical possibility of finer microtonic divisions, more accurately reflects the actual performance of traditional Eastern instruments [6].

Manzeli corresponds to "***house***" and similar terms, while ***makani*** corresponds to "***place***" and similar terms.

In 1405, Masih Aslani conducted research examining modern numerical systems in musical modes and their relationship with temperaments and constellations [26–32]. Also in 1405, the artist Daniel Bikas presented research in the field of art [33]. Fatemeh Etemadi-Rad analyzed research concerning art and tourism in 1405 [34–36]. In 1400, Mohsen Poursheikhi presented research regarding modern numerical systems and their connection to commodity production [37–38]. Batul Shahriyar analyzed the topics

of frostbite, growth, and temperament in 1399 [39–40]. With the assistance of Reza Kakoulvand, research investigating numerical systems in conjunction with computational and experimental fluid dynamics was conducted in 1401 [41–51].

Innovative numerical arrangements of musical scales and their calculation methods based on the common intervals of different scales in an octave, in twelve, seventeen, and twenty-four notes, will be introduced, which have been developed to determine the inherent and acquired temperaments and the relationship of the celestial zodiac constellation and the planets of each scale. This paper presents the first study to examine the complex relationship between the four human temperaments and the zodiac constellations with musical scales. While previous researchers acknowledged this connection, they failed to develop any logical, systematic, or computational framework to clarify it. Furthermore, their interpretations often differed significantly and left room for ambiguity [3]. In a calculus number system with 17 notes, the initial note "***Do***" and the final note "***Si***", the ascending sequence (in total, 17 notes or 16 consecutive unequal intervals) is represented in two models: place and house (symbolizing place and positional value) with the same order but different numerical values with common calculus rules. As shown in ***Table 1***, the order of notes, place numbers, and as shown in ***Table 2***, house numbers are based on valuation [3].

Table 1: Spatial Numerical Model 17

Note symbols	C	D♭	D♯	D	E♭	E♯	E	F	G♭	G♯	G	A♭	A♯	A	B♭	B♯	B
Note names	Do	Re♭	Re♯	Re	Mi♭	Mi♯	Mi	Fa	Sol♭	Sol♯	Sol	La♭	La♯	La	Si♭	Si♯	Si
Makāni numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Table 2: 17-note numerical home model

Note symbols	C	D♭	D♯	D	E♭	E♯	E	F	G♭	G♯	G	A♭	A♯	A	B♭	B♯	B
Note names	Do	Re♭	Re♯	Re	Mi♭	Mi♯	Mi	Fa	Sol♭	Sol♯	Sol	La♭	La♯	La	Si♭	Si♯	Si
Manzeli numbers	1	2	3	4	5	6	7	8	9	2	3	4	5	6	7	8	9

2. Research Objectives:

1. ***The main objective of the research:*** to introduce and validate a 24-tone numerical system along with its computational rules for the scientific analysis of music.

2. Sub-objectives:

• To establish a connection between musical scales and:
a) the four humors (***temperaments***) in traditional medicine
b) the constellations of the zodiac
• To present a new model that relates the following:
a) the four classical elements
b) celestial bodies with musical notes
• To test the scale symmetry criteria in the proposed system.

2. Introducing the 24-tone numerical system (a new theory)

First and foremost, it is necessary to emphasize that the proposed model has no previous precedent in Iranian and Western music theories for the specific application mentioned in this study. The 24-tone numerical system is a theoretical structure in which the octave is divided into 24 equal intervals, each representing a quarter note.

In this system, each note has a corresponding number, element, and planet, and has characteristics based on the influences of the elements and planets. One reason for the difference in the modes of the same scale with different keys is the elemental character and planetary nature of the "***witness***" or tonic of the 4th scale. For example, when a melody is played in D major, the listener experiences a different feeling than in F major. This is because, despite having the inherent temperament of the scale, the tonic note of D major is influenced by the planet Mercury and the tonic of F major is influenced by the planet Jupiter, as shown in the section below. This is the same for all scales and modes. Of course, the change in the mental states of a scale may be related to other reasons, such as "changing the number of commas in the intervals", the mental states of the performer, the gender of the witness (masculine, feminine, or neuter)5 and the sound quality of the musical instrument. 2.1. 24-Note Table Approach

To obtain the elements and planets of the notes, a 28-cell table should be used, since the product of four elements and seven planets is 28; however, since this table can accommodate only 24 notes, four empty spaces must first be identified. Therefore, a table with 24 cells should be designed, as shown in Table 5. Then the number 9 is given to the first cell of the table, and from then on, the number of each cell is multiplied by 9. In other words, nine is added to nine and the number obtained from each cell is divided by 4. The remainder is the element notation according to ***Table 3***.

Table 3: *Elements and corresponding notation[3]*

Element	Fire	Wind	Water	Earth
Notations	1	2	3	4

Table 4: Planets and their corresponding symbols

Planets	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Notations	1	2	3	4	5	6	7

Note: When the remainder is zero, it represents the

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month, but ancient people considered the number 7 to be its symbol.

Table 5: 24-part table for extracting elements and planets of interest

Row	1	2	3	4	5	6	7	8	9	10	11	12
Multiple9	9	18	27	36	45	54	63	72	81	90	99	108
Element	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth
Plt.Nos	2	4	6	1	3	5	0	2	4	6	2	3
Row	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Multiple9	117	126	135	144	153	162	171	180	189	198	207	216
Element	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth	Fire	Wind	Water	Earth
Plt.Nos	5	0	1	4	6	1	3	5	0	2	4	6

Therefore, according to the arrangement of the elements and planets, the 8 positions of the four empty houses are determined according to ***Table 6***.

Table 6: Position table of four empty cells

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	Empty	1	5	9	13	17	21
Wind	18	22	Empty	2	6	10	14
Water	15	11	19	23	Empty	3	7
Earth	4	8	12	16	20	24	Empty

Note: The numbers in Table 6 are the row numbers in Table 5.

The table above shows four empty cells, 8 cells apart. Therefore, the distance between each filled cell is 9 cells. Then, the note "A", which is the origin of the other notes, is placed in the 14th cell in the center of the 28-cell table, so that the point of unity of all the notes is located. Subsequently, the remaining notes in Table 8 (cell 28) are arranged according to the sequence specified in Table 7.

Table 7: Notes Order Table

Number	1	2	3	4	5	6	7	8	9	10	11	12
Note	La	La♯	Si♭	Si♭	Si	Si♯	Do	Do♯	Re♭	Re♭	Re	Re♯
Number	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Note	Mi♭	Mi♭	Mi	Mi♯	Fa	Fa♯	Sol♭	Sol♭	Sol	Sol♯	La♭	La♭

Note 1: A flat note (♭) lowers the pitch of the note by a semitone (half step).
Note 2: A sharp (♭) lowers the pitch of the note by approximately a quarter step.
Note 3: A sharp (♯) raises the pitch of the note by approximately a quarter step.

Table 8: Note positions in a 28-cell Makani structure

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	Empty	Fa	Sol	La♭	Si♭	Do	Re
	1	5	9	13	17	21	25
Wind	Mi♭	Fa♯	Empty	**La**	Si	Do♯	Re♯
	2	6	10	14	18	22	27
Water	Mi	Sol♭	Sol♯	La♯	Empty	Re♭	Mi♭
	3	7	11	15	19	23	27
Earth	Mi♯	Sol♭	La♭	Si♭	Si♯	Re♭	Empty
	4	8	12	16	20	24	28

If any note other than "***A***" (***La***) is placed in the central cell (***the 14th cell***) of the 28-cell chart, the result becomes unstable and the overall harmonic coherence of the musical system within the chart is destroyed. This instability serves as evidence that the note "A" acts as the only possible point of balance and forms the basis for determining the position of all other notes, so that the location of each note is defined relative to this fixed center.

Table 9: The 9-part table of 28-cell Makāni of notes

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	Empty	Fa	Sol	La♭	Si♭	Do	Re
	1	5	9	4	8	3	7
Wind	Mi♭	Fa♯	Empty	**La**	Si	Do♯	Re♯
	2	6	1	5	9	4	8
Water	Mi	Sol♭	Sol♯	La♯	Empty	Re♭	Mi♭
	3	7	2	6	1	5	9
Earth	Mi♯	Sol♭	La♭	Si♭	Si♯	Re♭	Empty
	4	8	3	7	2	6	1

Makani numbers are based on the order of the notes, while Manzeli numbers are based on the valuation [3]. Table 9 shows that every eight notes are separated by one interval, making a total of nine intervals. Table 10 also specifies the place numbers of each note. It should be noted that the author-researcher developed the basic logic of this number system through extensive trial and error, and tested numerous methods to derive them all, as mentioned earlier.

Table 10: Number of house notes in the 24-note system

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire		Fa	Sol	La♭	Si♭	Do	Re
		5	9	5	9	5	9
Wind	Mi♭	Fa♯		**La**	Si	Do♯	Re♯
	2	6		6	2	6	2
Water	Mi	Sol♭	Sol♯	La♯		Re♭	Mi♭
	3	7	3	7		7	3
Earth	Mi♯	Sol♭	La♭	Si♭	Si♯	Re♭	
	4	8	4	8	4	8	

The order of the note house tables is place numbers and, by value, house numbers. The logic governing house numbers in the 24-note system (similar to the 17-

note system) inherently avoids two-digit numbers through numerical reduction. The house numbering system operates on a 28-cell matrix divided into 9 sections, where the value of each cell is obtained through a cyclic digit addition process that systematically manages the occurrence of two digits. This work presents a new mathematical formulation of this algorithm - the cyclic boundary theorem (***CBT***) - which, to the best of our knowledge, has not been previously described in the literature. CBT provides a rule for managing numbers in repeating 9-digit cycles. When 9 (the end of the cycle) is reached, it becomes 10 → 2 (instead of the conventional 10 → 1), giving rise to predictable boundary behavior, as shown in ***Table 11***.

Cyclic Boundary Theorem (***CBT***)

A mathematical framework for periodic number systems defined as follows:

Notation:

• $k \in Z^+$: cycle index (each cycle consists of 9 consecutive numbers)

• $P_k = 9$: base value at the cycle boundary

House numbers (constant for all complete cycles)

• M_k: final computed value after transformations

• $dr(x)$: digital root of x (e.g., $dr(123) = 1 + 2 + 3 = 6$)

Basic algorithm:

$$M_k = \begin{cases} k & \text{if } k = 1 \quad \text{(first cycle)} \\ dr(P_k + 1) + 1 = 2 & \text{if } k \geqslant 2 \quad \text{(subsequent cycles)} \end{cases}$$

Key mechanism:

At each cycle boundary (when $P_k = 9$):

$9 \rightarrow 9 + 1 = 10 \rightarrow dr(10) = 1 \rightarrow 1 + 1 = 2 \rightarrow M_k = 2$

In other words, when reaching the house number 9 (the cycle boundary), the system calculates the initial value of the new cycle by successively adding the digits: first, it adds the digits 9 + 1 (since the tenth cell corresponds to the house number), then it adds 10 to 1, which results in 11, and the sum of its digits (1 + 1) is equal to 2.

Table 11: House number valuation in a 28-house matrix

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	1	Fa 5	Sol 9	La 5	Si 9	Do 5	Re 9
Wind	Mi 2	Fa 6	10+1=11 1+1=2	La 6	Si 10+1=11 1+1=2	Do 6	Re 10+1=11 1+1=2
Water	Mi 3	Sol 7	Sol 3	La 7	3	Re 7	Mi 3
Earth	Mi 4	Sol 8	La 4	Si 8	Si 4	Re 8	4

3. Features of the 24-note numerical system of the table

The originality of each table can be recognized by its structural features, which for the spatial and house tables of the 24-note numerical system are as follows:

1. The position of the note "La" in the spatial model of the 14th cell causes each of the seven natural notes to be assigned to a planet. In other words, there will be no planet with two natural notes.

The numbers of the note "***La***" in the 28-cell Makani tables of notes (number 14 in Makani table 8 and number 5 in Makani table 9) are in the hidden numerical structure, the same as the house number of the note "La", that is, the number 6. In the Makani model, the number "14", which is the number 5 in the sum of the components (1 + 4 = 5), is the same as the number of the nine-part Makani model. The number 5 also adds up to 15 in the sum of the natural numbers (1 + 2 + 3 + 4 + 5 = 15). Finally, the sum of the digits of the number 15, that is, (1 + 5 = 6), is equal to the house number of the note "***La***".

Prime number (***Table 8***): 14

Digital root of number 14: $dr(14) = 1 + 4 = 5$

$$\text{Sum of natural numbers } \left(\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\right) : \sum_{i=1}^{5} i = \frac{5 \times 6}{2} = 15$$

$$\text{Digital root of } 15 \ : dr(15) = 1 + 5 = 6$$

In a 9-part table, the empty cells are equally spaced. That is, when the cells are arranged around a circle, the first and last cells will have the same position. In this context, the sum of the digits obtained by adding up all the numbers in the table of 9, assuming the first and last cells are the same, is equal to 9, as shown in Table 12.

Table 12: Sum of numbers in a 9-part spatial model using the digital root

Sum of all numbers	Digital Root
$1+2+\cdots+9+1+2+\cdots+9+1+2+\cdots+9 = 135$	$dr(135) = 9$

The characteristic of the number 135 is that the remainder when divided by its constituent numbers (1, 3, and 5) is the numbers 9 and 0.

As shown in Table 13, the sum of the corresponding numbers from each house in Table 10 (which represent the house notes) is 136. Furthermore, the sum of the digits of 136 (1 + 3 + 6) is equal to 10, the sum of its constituent numbers is 1 (1 + 0). This numerical harmony shows that all musical notes have a single, unified origin.

Table 13: Sum of Manzeli Numbers

Sum of all numbers	Digital Root
$3 \times \left(\sum_{i=1}^{9} i\right) + 1 = 3 \times \left(\frac{9(9+1)}{2}\right) + 1 = 136$	$dr(136) = 10$ $dr(10) = 1$

5. The distance between the empty cells in the table of 9 is equal to nine. Therefore, for each empty cell, the number 1 is considered.

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By adapting the table of numbers to the system of letters of the alphabet, 9, the number of empty cells will be 1, 10, 100 and 1000, respectively, all of which will be equal to 1 without considering 0 (the sum of the digits will be 1).

6. In the table of houses of 10, by assigning the number 1 to the empty cells, the sum of the numbers in each column of the table will be a two-digit number, which can be reduced to a single-digit number by adding their digits, as shown in Table 10. Again, the number 1 is obtained by reducing the sum of the final two-digit or single-digit numbers, as shown in Tables 14 and 15.

Table 14: Numerical values in the 24- note house system

Names	Saturn	Jupiter	Mars	Sun	Venus	Mercury	Moon
Fire	1	5	9	4	8	3	7
Wind	2	6	1	5	9	4	8
Water	3	7	2	6	1	5	9
Earth	4	8	3	7	2	6	1
sum	10	26	17	26	16	26	15
components	1	8	8	8	7	8	6

Table 15: Digital root calculations for numerical components

Sum of all numbers	Digital Root
$10+26+17+26+16+26+15=136$	$dr(136)=10$ $dr(10)=1$
$1+8+8+8+7+8+6=46$	$dr(46)=10$ $dr(10)=1$

7. Looking closely at the common numbers of the four elements in the seven planets of the house numbers of the 10-house chart of the notes, it is clear that each element consists of two numbers. Fire (5 and 9), Wind (2 and 6), Water (3 and 7) and Earth (4 and 8). By subtracting the larger number from the smaller number, the number 4 is obtained, that is, (9 - 5 = 4) for Fire, (6 - 2 = 4) for Wind, (7 - 3 = 4) for Water and (8 - 4 = 4) for Earth, which is equal to the number of elements. 8. When the sum of the common numbers of each element of the 10-house chart of the notes is added together, it becomes 44 and the sum of its digits is equal to 8. Interestingly, the distance between each empty cell of the 28-house chart is also 8 cells. This is illustrated in Table 16 using the fact that
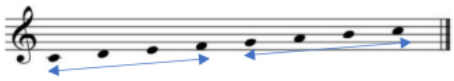
Fire (5 + 9 = 14), Wind (6 + 2 = 8), Water (3 + 7 = 10), and Earth (4 + 8 = 12).

Table 16: Sum of the numbers of the four elements

Sum of all numbers	Digital Root
$12+10+8+14=44$	$dr(44)=8$

4. Symmetric and Asymmetric Scales Based on 24-Note Numbering Systems

As was shown in a previous study [3], when analyzing symmetric and asymmetric scales, the structural similarity between two tetrachords alone is not sufficient for classification. It is important that the sum of the calculated note values using the 24-note house numbering system also be the same for both tetrachords. This requirement takes into account cases where apparent structural similarities may result in different numerical values. The classification models are presented in Figure 1 with Table 17 (for symmetric scales) and in Figure 2 with Table 18 (for asymmetric scales) following this procedure.
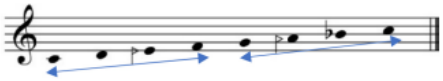


W, W, H- W -W, W, H

Figure 1: Mahur - C (Do)

Table 17: Symmetrical scale, Māhūr - Notes

C	D	E	F	Notes	G	A	B	C
5	9	3	5	Numbers	9	6	2	5
$5+9+3+5=22$				Total	$9+6+2+5=22$			



W, M, M- W -M, M, W

Figure 2: c: Segāh - $E_{\flat}$ ($Mi_{\flat}$) - *tone*

C	D	$E_{\flat}$	F	Notes	G	$A_{\flat}$	$B_{\flat}$	C
5	9	2	5	Numbers	9	5	8	5
$5+9+2+5=21$				Total	$9+5+8+5=27$			

Table 18: Asymmetrical scale, Segāh- Notes

Note 1: "M" means "***majnabba***" = half note ± quarter note (ascending or descending).

The importance of specific notes in a scale requires special attention. An important point to emphasize is:

The witness note: The witness note, which acts as the central melodic piece, acts as the main tonal support to which musical phrases are continuously attached [15]. It is noteworthy that these witness notes are systematically determined based on the ascending degrees of the corresponding scales [6]. Accordingly, the witness notes of seven Iranian musical instruments are presented in

Table 19.

Table 19: Scale Mode Specifications[3]

Number	Mode	Shāhed note
1	Māhūr	Tonic
2	Rāst-Panjgāh	Tonic, then Supertonic
3	Chāhārgāh	Tonic
4	Segāh	Mediant
5	Shūr	Supertonic
6	Navā	Supertonic
7	Homāyun	Supertonic

Note: The degrees of the modes in Table 19 are based on their intrinsic scale, not their practical or acquired scale.

5. Computational Notation

The notations used in this article are presented in Table 20.

Table 20: Computational Notation of the 17-Note Numeral System

Explanation	Notation
Symmetrical scale with a Shāhed note	St_1
Symmetrical scale with two Shāhed notes	St_2
Asymmetric scale with a Shāhed note	$\tilde{St}_1$
Sum of the notes	Sn_7
First Shāhed note	t_1
Second Shāhed note	t_2
The first tetrachord	x_1
The second tetrachords	x_2
The difference in the sum of the Manzeli numbers of two tetrachords	X
Extracted number	e
Answers to calculations in the first, second	a_1, a_2
Obtained number from total calculations	TC
Final innate element of scale (temper notation)	Tes
Scale zodiac constellation	Sz

6. Computational rules of the 24-note number system.

To obtain the element number and the zodiac constellation of the scale, the same computational approach is used as in the 17-note number system. However, the method of obtaining the extracted number is different. It should be noted that with the help of the old system in cases ff and III below and with a tedious process of trial and error calculation in cases I and IV, the following rules have been obtained:

1. The result of the calculations in both numerical music systems (17 and 24 notes) should be the same.

2. Obtaining the corresponding scale element (natural temperament step): Divide the last number in the whole calculation by 4 and the remainder will be the scale element, i.e.

TC/4= Tes.

where the notation a/c≡ b means that b is the remainder of dividing a by c.

Each element is also associated with a temperament. The numerical symbol of each element and their temperaments are shown in Table 21 [2].

Table 21: Numerical symbols of each element and their temperaments [2]

Element	Fire	Wind	Water	Earth
Temper	Warm and dry	Warm and moist	Cold and moist	Cold and dry
Notation	1	2	3	4

Table 22: Numerical symbols of the zodiac constellations[3].

Zodiac	Aries	Taurus	Gemini	Cancer	Leo	Virgo
Notation	1	2	3	4	5	6
Zodiac	Libra	Scorpio	Sagittarius	Capricorn	Aquarius	Pisces
Notation	7	8	9	10	11	12

Note: When the remainder is zero, "Sz" represents Pisces, but the ancients considered the number 12 as its symbol.

IV. The instructions for the extracted number in the 24-note number system for symmetrical and asymmetrical scales are given below. Note that the extracted number is needed to accurately calculate the zodiac constellation in the scale.

• ***Symmetric scale:*** The extracted number (e) is not needed. Because the difference (X) between the sum of the house numbers of the two tetrachords in the scale will be zero.

$$X := x_2 - x_1 \quad (\text{where } X = 0)$$
$$e := X \quad (\text{thus, } e = 0)$$

• ***Asymmetric scale with a note:*** 12 The difference between the sum of the house numbers of two tetrachords of a scale (X) is calculated as follows:

– If $X = |x_2 - x_1| > 4$, repeatedly reduce X by 4 units until it is in the range

$1 \leqslant X \leqslant 4$ (i.e.X mod 4).

– The final extracted number (e) is obtained by multiplying the result by 4.

$$X = |x_2 - x_1|, \quad e := 4 \times \begin{cases} X \bmod 4 & \text{if } X > 4 \\ X & \text{otherwise} \end{cases}$$

Note: The difference (X) must be between 1 and 4. If X > 4, subtract 4 repeatedly until the result is within this

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range (e.g., if X = 11, then 11 − 4 − 4 = 3, and e = 4 × 3 = 12).

7. Calculation method of the 24-note number system
Before discussing the calculation method, it is worth noting that the author arrived at this number system and calculation method through trial and error and testing of various methods.

1. Seven notes are counted in the total notes of the step and there is no need for the Sn7 octave.

2. In symmetrical and asymmetrical steps that have one witness note (the calculation method is the same),

the sum of the seven notes of a step is multiplied by the number of witness notes; but the final answer

in asymmetrical steps is added to the extracted number as shown in the following figure:

A. Symmetric step with one witness note
$TCSt_1 := Sn7 \times t1.$
B. Asymmetric step with one witness note

$TC\ddot{S}t_1 := (Sn_7 \times t_1) + e.$

3. Symmetric step with two witness notes.
A. The sum of the scale numbers (which are seven notes) is multiplied by the first witness note and is represented as
$a_1 := Sn_7 \times t_1.$
B. The second witness note is multiplied by 4 and added to the result of step "A"
$TCSt_2 := (t_2 \times 4) + a_1.$
Note: As stated in Article 17-Note in the Sixth Appendices, "twelve is a complete cycle or the end of a repetition"
And the number 24 also has the same property in repetition. Therefore, the 12-note and 24-note systems are the same, but the 17-note system is an incomplete cycle, so its calculations will be different compared to the other two systems in two respects. 1. Calculation of the octave number in the total number of notes for each scale (in the 24-note and 12-note systems, 7 notes and in the 17-note system, 8 notes are calculated by taking the octave number into account).

2. The calculations of the scale elements (es_1 and es_2), although irrelevant in the 12- and 24-note systems, become necessary to resolve the incomplete cycle inherent in the 17-note system.13

2.Result and Discussion:

8. Findings:

Mahur - C (Do): Symmetrical scale with a witness note as shown in Table 23.

Table 23: The reference note is C=5

C	D	E	F	G	A	B	Sn_7
5	9	3	5	9	6	2	39
$TC = 39 \times 5 = 195.$							

A: The remaining number is 3 and its temperament is cold and damp.

195/4≡ 3
B: The remaining number is 3 and the scale is the constellation Gemini.

195/12≡ 3
Chāhārgāh- C (Do): Symmetrical scale with the witness note as given in Table 24.

Table 24: The Shahed note is C =5

C	$D_\flat$	E	F	G	$A_\flat$	B	Sn_7
5	8	3	5	9	5	2	37
$a_1 = 37 \times 5 = 185.$							

A: The remaining number is 1 and the temperament is hot and dry.
185/4≡ 1
B: The remaining number is 5 and the scale is the constellation of Leo.
185/12≡ 5
Right Panjgah - Do (C): A symmetrical scale with two witness notes as shown in Table 25.

Table 25: The control notes are C=5 and D=9.

C	D	E	F	G	A	B	Sn_7
5	9	3	5	9	6	2	39
$a_1 = 39 \times 5 = 195,$ $a_2 = 4 \times 9 = 36,$ $TC = 195 + 36 = 231.$							

A: The remainder is 3, and its temperament is cold and moist.
231 ÷ 4
Remainder: 3
B: The remainder is 3, and the corresponding constellation (or degree) is Gemini.
231 ÷ 12
Remainder: 3
Segah – Middle degree - $E_\flat$ ($Mi_\flat$): An asymmetrical scale with a *shahed* (witness/pivot) note, as shown in Tables 26 and 27.

Table 26: The extracted number of the scale's Segāh - $E_{\flat}$ *& Notes*

C	D	$E_{\flat}$	F	Notes	G	$A_{\flat}$	$B_{\flat}$	C
5	9	2	5	Numbers	9	5	8	5
5+9+2+5=21				Total	9+5+8+5=27			
27-21=6,		6-4=2,			$e = 2 \times 4 = 8.$			

Table 27: The Shāhed note is $Mi_{\flat}$ =2 *& TC*

C	D	$E_{\flat}$	F	G	$A_{\flat}$	$B_{\flat}$	Sn_7
5	9	2	5	9	5	8	43
$a_1 = 43 \times 2 = 86,$							
$TC = 86 + 8 = 94.$							

A: The remainder is 2, and its temperament is hot and moist.
94 ÷ 4
Remainder: 2
B: The remainder is 10, and the corresponding constellation is Capricorn (the Goat).
94 ÷ 12
Remainder: 10.

Homayun - $A_{\flat}$ ($La_{\flat}$): An asymmetrical scale with ****shahed**** (focal) notes, as shown in Tables 28 and 29.

Table 28: The extracted number of the scale's Homāyun - $A_{\flat}$ *- Notes*

G	$A_{\flat}$	B	C	Notes	D	$E_{\flat}$	F	G
9	5	2	5	Numbers	9	3	5	9
9+5+2+5=21				Total	9+3+5+9=26			
26-21=5,		5-4=1,			$e = 1 \times 4 = 4.$			

Table 29: (Manzeli) - The Shāhed note is $A_{\flat}$ =5

G	$A_{\flat}$	B	C	D	$E_{\flat}$	F	Sn_7
9	5	2	5	9	3	5	38
$a_1 = 38 \times 5 = 190,$							
$TC = 190 + 4 = 194.$							

A: The remainder is 2, and its temperament is hot and moist.
194 ÷ 4
Remainder: 2
B: The remainder is 2, and the corresponding zodiac sign is Taurus.
194 ÷ 12
Remainder: 2
Nava - A (LA): An asymmetrical scale with a ****shahed**** (dominant/reference) note, as presented in Tables 30 and 31.

Table 30: Extracted value for Nova-A step

G	A	B	C	Notes	D	E	F♯	G
9	6	2	5	Numbers	9	3	6	9
9+6+2+5=22				Total	9+3+6+9=27			
27 -22=5,		5 -4=1,			$e = 1 \times 4 = 4.$			

Table 31: The result for control A is 6.

G	A	B	C	D	E	F♯	Sn_7
9	6	2	5	9	3	6	40
$a_1 = 40 \times 6 = 240,$							
$TC = 240 + 4 = 244.$							

A: The remainder is 0, and its temperament is cold and dry.
244 ÷ 4 = ... with a remainder of 0.
B: The remainder is 4, and the zodiac sign associated with the scale is Cancer (the Crab).
244 ÷ 12 = ... with a remainder of 4.
Shur – D (Re): An asymmetrical scale with a ****shahed**** (witness) note, as shown in Tables 32 and 33.

Table 32: Extracted value for the Shūr-D scale

C	D	$E_{\flat}$	F	Notes	G	A	$B_{\flat}$	C
5	9	2	5	Numbers	9	6	8	5
5+9+2+5=21				Total	9+6+8+5=28			
28-21=7,		7-4=3,			$e = 3 \times 4 = 12.$			

Table 33: (Manzili) – The witness note D is 9.

C	D	$E_{\flat}$	F	G	A	$B_{\flat}$	Sn_7
5	9	2	5	9	6	8	44
$a_1 = 44 \times 9 = 396,$							
$TC = 396 + 12 = 408.$							

A: The remainder is zero, and its temperament is cold and dry.
408 ÷ 4 ≡ 0
B: The remainder is zero, and the corresponding constellation is Pisces (the Fish).
408 ÷ 12 ≡ 0
The results of the calculation steps for the seven systems are presented in Tables 34 through 36.

Table 34: Scale symmetrical about the reference note

Mode	Sn_7	t_1	TC	Tes	Sz
Māhūr	39	5	195	3	3
Chāhārgāh	37	5	185	1	5

Table 35: Symmetrical scale with two reference notes

Mode	Sn_7	t_1	a_1	t_2	a_2	TC	Tes	Sz
Rāst-Panjgāh	5	39	195	9	36	231	3	3

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Table 36: Asymmetrical scale relative to the tonic note

Mode	t_1	Sn_7	e	a_1	TC	Tes	Sz
Segāh	2	43	8	86	94	2	10
Homāyun	5	38	4	190	194	2	2
Navā	6	40	4	240	244	0	4
Shūr	9	44	12	396	408	0	0

9. Discussion

Music is a significant factor influencing the balance of the four psychological characteristics associated with human temperaments; consequently, understanding temperament and the effects of musical modes (**maqāms**) offers a novel avenue for purposeful research aimed at addressing physical ailments and psychological disorders.

In their writings regarding the relationship between music, temperaments, and celestial signs (zodiac signs), Iranian scholars have highlighted specific ***maqāms*** and their effects. However, two challenges persist in this field: first, the absence of a defined formula or rule to explain this phenomenon has led some to dismiss these perspectives as mere products of imagination [16]. Second, verifying these views is difficult due to inconsistencies in the nomenclature of musical scales and modes, as well as uncertainties regarding contemporary Iranian musical practices [17]. Therefore, the present article—along with related studies on numerical systems for musical scales [3]—represents a pioneering effort to investigate this important subject through the numerical coding and calculation of musical notes.

Innovative numerical systems in music employ an interdisciplinary approach to elucidate the psychological effects of music. By utilizing mathematical analysis of musical structures, these systems establish a basis for correlating musical elements with temperamental traits (based on the theory of the four humors) and astrological characteristics (zodiac signs). Given the established psychological associations within humoral medicine and astrology, this framework enables the extraction of patterns capable of predicting and explaining the psychological impact of music. Empirical details and practical applications of this model will be presented in future publications. Innovative numerical systems for musical scales and their governing rules represent a fresh approach to understanding theoretical relationships in music therapy, grounded in logical and numerical reasoning and calculations. This method employs numbers and calculations to express results; consequently, many subjective opinions based on personal perspectives can be refined and adjusted [3]. The 24-note system is more comprehensive than the 17-note and 12-note systems, and its existence is essential for validating and verifying the computational method.

Furthermore, there are significant points that must be considered; some were outlined in the article regarding the 17-note numerical system [3], while others are as follows:

1. Without access to the three calculation tables (two 17-note models and one 24-note "***Manzeli***" model), determining the calculation method for steps involving quarter-tone intervals is prone to error.
2. The calculation rules for the 24-note system differ slightly from those of the 17-note system, and calculations within each system are evaluated based on that system's specific methodology.
3. Note naming in both systems aims to facilitate understanding and align the notes with recognized intervals; however, the precise frequency position of the notes can undoubtedly vary depending on the mode or melodic context. In other words, the interval defined in terms of commas, savarts, or cents for a whole step is not constant; it varies according to the aural culture of different nations and ethnic groups. Consequently, the "equal temperament" system is unsuitable for monodic, melodic, and modal music (such as Iranian music), as melodies performed this way tend to sound sterile and lifeless. Exceptions may be made for certain types of polyphonic music, where some nuances might be sacrificed. For a skilled soloist, precise attention to the tuning (intonation) of notes and intervals is vital; even the slightest variation in a note's pitch within a given musical mode (***gusheh*** or ***maqam***) is of great significance.

For a master musician performing a solo, precise control over the sound of notes and intervals—as well as the slightest variation in pitch within any mode or ***maqam***—is crucial. For example, the note "***Mi***" in the fundamental mode of "Mahur-e Do" (C-Mahur) is approximately one *comma* lower than the "Mi" in the natural major scale. This results in an interval of approximately five ***commas*** between the notes "Mi" and "***Fa***"; however, this value is not factored into the calculations because the interval between "***Mi***" and "***Fa***" is inherently diatonic.

Musical intervals specific to a nation are defined by specific values (differences in ***commas***); in calculations, the frequency closest to the established standard is used, because the calculation itself deals with numerical values, whereas the issue of minute differences—such as those involving ***commas***—pertains to the realm of perception.

5. In practical application, the 24-note system does not require the use of frets spaced extremely close together; rather, it provides more precise data for calculating the position of each note for specific purposes. Consequently, the presence of every possible fret on string instruments is not essential. Furthermore, the 24-note system establishes a vital theoretical foundation for

microtones. By providing such a robust framework, it challenges the traditional Western musical paradigm—limited to whole tones and semitones—and paves the way for the practical application of microtones in fields such as music therapy, where their potential remains largely untapped.

6. In the 24-note system chart, each note is assigned an element representing its temperamental quality; this is achieved by basing the calculations on the frequency of the note "***A***" (standard pitch).

7. The intrinsic scale reflects mental states; for instance, the mental state associated with the fundamental mode of ****Dastgah-e Mahur**** (equivalent to the natural major scale) embodies qualities of dignity and tranquility. Dignity and stability are considered characteristics of the cold-and-moist temperament. However, the influence of the Gemini (***Twins***) constellation—which is hot and moist—fosters courage, a trait associated with the hot-and-moist temperament. Naturally, this holds true only when rhythm is not a factor and the melody is performed in free meter, as the mood (or character) of the rhythm significantly influences the mood of the melody.

8. Analyses in this study revealed that the absence of ascending quarter-tone intervals (****sori****) in the 17-note system [3]—compared to the 24-note system—prevents the full identification of the intrinsic scale of certain Iranian musical modes across various degrees and tonalities, unless a method and framework for comparing results from both systems is established. For instance, in the mode of "***Navā's Mi***," the absence of the note "***Do***" (C), and in "***Navā's La***," the absence of the note "***Fa***" (F), renders a precise comparison between the 17-note and 24-note systems impossible.

Consequently, the calculation of the principal mode of ****Dastgāh-e Navā**** in the article concerning the 17-note system [3] is incorrect, as there is no comparative framework or method linking it to the 24-note system. Although the argument presented in the 17-note system article regarding the modes of ****Navā**** and ****Segāh****—namely that "two scales within the same zodiacal constellation are harmonious"—applies to ****Navā**** and ****Bayāt-e Tork**** as well, more rigorous studies indicate that the intrinsic scale of ****Navā**** possesses a cold and dry temperament and is associated with the constellation Cancer (the Crab). This aligns with the emotional nature of ****Navā****; the ****dastgāh**** is suitable for conveying counsel and warnings in a calm tone, fitting the cold and moist temperament attributed to the constellation Cancer. The melancholic and introspective nature of this ****dastgāh**** is strongly linked to its intrinsic cold and dry temperament.

9. Subsequently, following an examination of the calculations regarding the ****Navā**** mode in the 24-note system, the corresponding calculations for the 17-note system are presented. A vital principle explained here—supported by a logical rationale—concerns quarter-tone notes (****sori****), which are absent from the 17-note system. Since the 17-note system tables represent an incomplete model of the 24-note system and lack specific numerical values for *sori* notes, the rule for determining their numerical value is as follows: the numerical value of the note preceding the *sori* is subtracted from the numerical value of the note following it (a calculation that typically yields the number 1); this difference constitutes the numerical value of the *sori* note. It should be noted that, in the final step, a constant value of 4—serving as a "normalization constant"—is added to the resulting figure.

Note: To date, this rule has been applied only once, in the analysis of ****Dastgāh-e Navā****. To verify its accuracy and generalizability, or to determine if any modifications are required, further testing and evaluation are necessary in future research involving similar musical scales—such as those presented in Tables 37 and 40.

Nava-La (A): An asymmetrical scale with a ****shahed**** (pivot/dominant) note, as presented in Tables 37 and 40.

Table 37: (Spatial) – Control result A = 14.

G	A	B	C	D	E	F♯	G	Total
11	14	17	1	4	7	1	11	66

$a_1 = 66 \times 14 = 924, \quad es_1 : 924 \overset{4}{\equiv} 4, \quad a_2 = 4 \times 4 = 16,$
$TC = 924 + 16 + 8 + 4 = 952.$

Table 38: (Manzali) – The witness note is A (La) = 6.

G	A	B	C	D	E	F♯	G	Total
3	6	9	1	4	7	1	3	34

$a_1 = 34 \times 6 = 204, \quad es_1 : 204 \overset{4}{\equiv} 4, \quad a_2 = 4 \times 4 = 16,$
$TC = 204 + 16 + 8 + 4 = 232.$

A. The remainder is 0, and its temperament is cold and dry.
952 ÷ 4
Remainder: 0
B. The remainder is 4, and the corresponding constellation is Cancer (the Crab).
952 ÷ 12
Remainder: 4

10. As shown in Figure 3, the scale of the ****Maqam**** "***Oshshaq***" is based on the fifth degree of the C major scale,
which serves as the fundamental key (mode) of the ****Dastgah**** Mahur. By adding a quarter-tone (****sori****) to

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the seventh degree
of this *Oshshaq* scale (in the key of G), the principal scale of the *Maqam* "Nava" is formed. This finding bears a strong resemblance
to the formation of the harmonic minor scale in Western music.

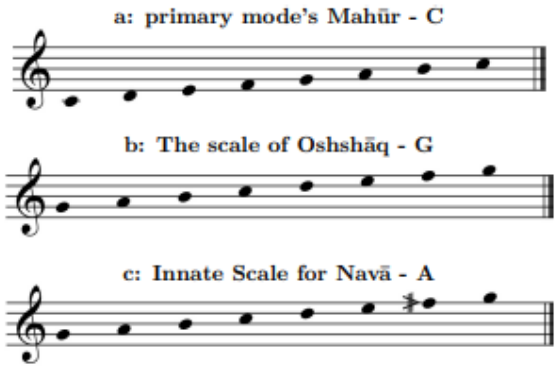


Figure 3: The order of emergence of the principal modal melodies up to approximately...

Although the 17-note and 24-note numerical systems share the same objective, the 24-note system is more comprehensive for the following reasons:
1.The 24-note numerical table allows access to knowledge regarding the temperaments and planets associated with the notes, whereas this is not possible with the 17-note table.

2. The realization of the principle of note simultaneity—in both consonant and dissonant contexts—can be mathematically demonstrated within the 24-note system, whereas no such rule exists in the 17-note system (this matter will be elaborated upon in future articles).
Research also indicates that the note "La" serves as the fundamental reference point, with the positions of all other notes measured and calibrated relative to it. The two systems share certain characteristics regarding the note "*La*"—considered the most important note—including:
1. In both systems, the fourteenth position (*Cell 14*) is the location of the note "*La*."
2. In both systems, the "*Manzeli*" value for the note "La" is 6. (The note "La" is also the sixth of the seven principal notes.)

10. Conclusion:

To validate and confirm the research findings concerning the 17-note system [3], the *Radif* of Iranian music was re-analyzed using the 24-note system. Computational results demonstrated that, despite fundamental differences in the numerical values of the notes and the structural division of the octave, the 24-note system and its associated formulas yield results that align perfectly with those of the 17-note system. This consistency is also evident in the assessment of the principal modal scales of the seven *Dastgahs* of Iranian music; the temperamental nature (intrinsic temperament) and the zodiac sign associated with each scale are accurately identified. Consequently, the significant numerical differences between the two systems do not affect the final analytical outcomes. This finding corroborates the research of Aslani and Karimi (*2018*); A study reported correlations between human temperaments and musical preferences within the framework of Persian musical modes (*dastgāhs*). Specifically, the researchers observed that individuals with a "cold and dry" temperament gravitated toward the *Shur* mode; those with a "hot and dry" temperament preferred the *Chahargah* mode; and those with a "cold and moist" temperament favored the *Mahur* mode [5].

The results indicate that, through calculations based on 17-tone and 24-tone systems, it is possible to determine the intrinsic temperament of any musical scale and examine its correlation with zodiac signs. As shown in Table 41, each zodiac sign possesses a unique temperament that may differ from the intrinsic temperament of its associated musical scale. This discrepancy gives rise to distinct interactions between the zodiac sign and the scale's temperament. While the scale's intrinsic temperament shapes the psychological states evoked by the melody, the temperament of the zodiac sign adds further layers of influence. When the temperaments of the zodiac sign and the scale align, their combined effect on the overall temperament is intensified, fostering harmonious mental states and a deeper impact on one's temperament.

Table 39: Table of innate temperaments and constellations (zodiac signs) in the seven *dastgāhs* of classical Iranian music.

Mode	Innate temperament	Zodiac sign	zodiac temperament
Homâyun	Warm and moist	Taurus	Cold and dry
Mâhûr	Cold and moist	Gemin	Warm and moist
Râst-Panjgâh	Cold and moist	Gemini	Warm and moist
Châhârgâh	Warm and dry	Leo	Warm and dry
Segâh	Warm and moist	Capricorn	Cold and dry
Navâ	Cold and dry	Cancer	Cold and moist
Shûr	Cold and dry	Pisces	Cold and moist

Despite superficial similarities in the intervals between the *Nava* and *Shur* *dastgahs* (musical modes), studies indicate that their fundamental modes are entirely distinct. For traditional musicologists, this distinction has long been the primary reason for regarding *Nava* as independent of *Shur*, thereby resolving doubts among some Iranian musicians regarding whether *Nava* shares a common origin with *Shur* or is derived from it. Analyses suggest that while structural similarities between tetrachords might imply symmetry, precise classification requires equality

in the sum of "***Manzeli***" values within the 24-note system. This numerical criterion reveals fundamental differences between symmetrical scales (characterized by equal tetrachord sums) and asymmetrical scales (with unequal sums), offering a precise, quantitative method for distinguishing scale types that goes beyond mere structural observation.

As a novel finding, the model correlating elements and planets with each note explains the consonant (***harmonious***) or dissonant (discordant) nature of notes—a topic to be addressed in the next article—as well as the reasons for the differing moods of identical scales played in different tonalities. Furthermore, understanding the relationship between elements and constellations via the 24-note numerical table—grounded in established rules and assessments—will prove beneficial for future research in therapeutic fields.

This study formulates a comprehensive mathematical model of the "***Manzeli***" four-cycle numbering system using a 28-cell matrix, revealing fundamental insights into cyclic modular arithmetic. The core innovation of this system lies in its automated "***digit reduction***" mechanism—a process triggered when intermediate sums reach 10, utilizing a nested digit-summation procedure (e.g., 2 = *dr(1 + dr(1 + 9))*). This boundary-sensitive transformation embodies a profound computational principle. The approach ensures three fundamental properties: (1) cyclic consistency through the precise preservation of periodicity, (2) deterministic transitions driven by predictable modular arithmetic at boundary points, and (3) emergent simplicity, wherein complex patterns reliably reduce to single-digit outcomes. Specifically implemented within the "***24-Note System***," this method (***CBT***) provides a unified computational framework for simultaneously managing conventional cyclic patterns and exceptional boundary behaviors; it holds direct applications in cryptography and discrete dynamical systems—domains where phase-transition control is critical. The elegance of this theorem stems from its dual capacity to maintain overall system stability while allowing for the redefinition of local rules at cycle boundaries. The "Cyclic Boundary Theorem" (***CBT***) offers two mathematically equivalent formulations. The primary version explicitly demonstrates the musically significant "***9-to-2***" transition via a fixed operational sequence, providing clear pedagogical insight into boundary-reset mechanisms. In contrast, the generalized version employs cyclic indexing (parameterized by cycle length) to encompass arbitrary periodic systems. Both formulations guarantee convergence to a single-digit number through the iterative calculation of the "digital root" and maintain definitive boundary transitions via controlled digit reduction. The structural clarity of the original formulation makes it particularly valuable for analyzing Manzeli's 28-cell system; conversely, the generalized version extends the theorem's applicability to diverse fields—including discrete mathematics (graph theory, modular arithmetic), cryptography, and dynamical systems—and demonstrates the framework's compatibility with position-dependent transformations in abstract algebraic structures. The generalized form is stated as follows:

Let $k \in Z+$ be the cell's positional index in equation.

- $n(k)$ indicates the cycle number containing the cell k in equation.
- M_k represents the final Manzeli number for cell k, computed through cycle-based digital root transformation in equation.
- The ∞ symbol in dr indicates iterative digital root computation until a single-digit number is obtained in equation.

$$n(k) = \left\lfloor \frac{k-10}{8} \right\rfloor + 1.$$

$$M_k = \begin{cases} k & \text{if } 1 \leqslant k \leqslant 9, \\ dr^{\infty}(k + n(k)) & \text{if } k \geqslant 10. \end{cases}$$

This research not only provides scientific validation for historical hypotheses regarding the connections between music, mathematics, tuning theory (temperament), and astronomy, but—through its precise computational models—also establishes a foundation for interdisciplinary research in the fields of music psychology, music therapy, and cultural studies. Thanks to its enhanced precision in representing musical intervals, the proposed system will serve as an effective tool for future research into the profound effects of music on the human psyche and physiology.

11. Appendices

I. Zodiac constellations and their corresponding planets: Each of the twelve segments of the zodiac—through which the sun passes each month—roughly corresponds to the solar months. Each zodiac sign has a corresponding constellation (planet) as follows: 1- Aries: Mars; 2- Taurus: Venus; 3- Gemini: Mercury; 4- Cancer: Moon; 5- Leo: Sun; 6- Virgo: Mercury; 7- Libra: Venus; 8- Scorpio: Mars; 9- Sagittarius: Jupiter; 10- Capricorn: Saturn; 11- Aquarius: Saturn; 12- Pisces: Jupiter.

II. It is assumed that a smaller circle is situated within a larger circle (both concentric) and divided into 12 equal parts (see Figure 4). The large circle represents equal semitone intervals, while the small circle represents equal quarter-tone intervals; This results in a total of 24 points within the 360-degree angular space. Since the base number used to derive the 24-part table of elements and planets must be a single-digit number, the number 360 is reduced to a single digit by summing its digits (3 + 6 + 0 = 9).

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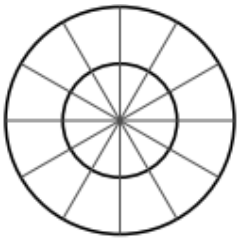


Figure 4: Diagram of nested circles showing 12-fold symmetry.

It appears that the ancients associated the number 9 with music. They divided the interval of a whole step into nine equal parts, terming this unit a "***comma***."
Furthermore, based on Table 40 and the system of Latin letter-value assignments (Gematria), the numerical value of the word "***comma***" is 144—the digits of which sum to 9. This is demonstrated below through the numerical values of the letters in the Latin alphabet Gematria table, confirming the validity of this reasoning (as shown in ***Table 41***).

Table 40: Correspondence of letters and numbers

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Number	1	2	3	4	5	6	7	8	9	10	20	30	40
Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Number	50	60	70	80	90	100	200	300	400	500	600	700	800

Table 41: Letter values and calculations

A	M	M	O	C	Letter Sum	Digit Sum
1	40	40	60	3	$3+60+40+40+1=144$	$\mathrm{dr}(144)=9$

III. Innate scale: A scale whose interval structure—spanning from a specific degree to its octave—is identical to that of other scales of the same type, thereby defining it as the fundamental (innate) form relative to them. Consequently, in a 17-note system, a single scale may be presented in 17 different modal rotations, yet only one of these 17 interval patterns serves as the reference (innate) scale [3].
IV. Acquired scale: A scale that begins on a different degree but retains the same interval structure as the innate scale. For example, the acquired "Mahur" scale on the note "Fa" is obtained by applying the interval pattern of the "***innate Mahur***" (which is based on "Do") but starting from the note "***Fa***" [3].
V. Practical scale: A scale where the specific note from which it is performed determines the character and emotional mood of that **maqam**. For instance, the natural minor scale (A minor) is derived from the natural major scale (C ***major***); however, the practical "***A minor***" scale—the scale as it is actually performed—begins on the sixth ascending degree of the "C major" scale, and it is this specific starting point that establishes the identity of that **maqam** [3].

3.References:

[1] ناشناس. (912). نسخه خطی «اخوان‌الصفا» یا «مجمل‌الحکمه».

[2] Aslani, M. (2025). An innovative numerical system model to identify the effects and psychological states of musical rhythm and tempo through four temperaments. Mathematics and Computational Sciences, Articles in Press. https://doi.org/10.30511/mcs.2025.2044621.1252 1, 21, 19.

[3] Aslani, M. (2025). Innovative 17-note number system and its computational method to identify the effects and psychological states of music scale. Transactions on Combinatorics (TOC). https://doi.org/ 10.22108/toc. 2025.145560.2290 1, 3, 3, 1, 1, 10, 19, 1, 22, 13, 1, 18, 1, 1.

[4] اصلانی، م. (1397). رابطه میان چهار مزاج با ترجیح موسیقی سنتی ایرانی در افراد 30 تا 45 ساله: ارائه مدل (پایان‌نامه منتشرنشده کارشناسی ارشد). دانشگاه آزاد اسلامی، واحد علوم و تحقیقات، تهران، ایران.

[5] Aslani, M. & Karimi, B. (2018). The relationship between four temperaments with the preference mediation of Traditional Iranian music in people ranging 30 − 45 years Old: providing model. Medical History Journal, 10(367–23.https://doi.org/10.22037/mhj.v10i36.23279 1

[۶] برکشلی، م. (۱۳۴۲). شرح ردیف موسیقی ایران، هفت دستگاه موسیقی سنتی ایران (ردیف). (به کوشش م. معروفی). اداره کل هنرهای زیبای کشور. ۱، ۴، ۱، ۲۰.

[7] چغمینی خوارزمی، م. ب. م. (2011). در طب (ترجمه: ا. ناظم). انتشارات المعی. (اثر اصلی در دوران قرون وسطی منتشر شده است).

[8] دهدار، م. (حدود ۱۵۰۰). نسخه خطی کتاب جفر.

[9] فخرالدینی، ف. (1400). تحلیل و شرح ردیف موسیقی ایران (ویرایش هفتم). انتشارات معین.

[10] فارابی، ا. ن. (۲۰۱۲). *موسیقی کبیر* (ترجمه آ. آذرنوش). پژوهشگاه علوم انسانی و مطالعات فرهنگی. (اثر اصلی نگارش‌یافته در حدود سال ۹۵۰).

[11] هرگنهـان، بـی. آر. (2015). تـاریخ روان‌شناسـی (ترجمـه ی. سیدمحمدی؛ ویرایش چهارم). انتشارات ارسباران. (اثر اصلی پـیش‌تـر منتشر شده است).

[12] Ibn Voshmgir, A. M. S. E. M. N. Q. (c. 1600b). Bahjat al-rūḥ (Ouseley 117, 1b-23). Bodleian Library, Oxford.

[13] مشحون، س. هـ (1380). تاریخ موسیقی ایران (ویـرایش اول). انتشارات فرهنگ نشر نو.

[14] مرادیان، ه. (۱۳۹۶). ردیف موسـیقی ۲: موسـیقی نظـری ایـران (ویرایش اول). تهران.

[15] Miller, L. (2012). Music and Song in Persia (RLE Iran B): The Art of Avaz. Routledge.

[16] ناظم‌پور، م. (۱۳۹۱). کاربرد موسیقی در هیپنوتیزم و مدیتیشن (ویرایش دوم). انتشارات ساوالان.

[17] ناصـری، م.، رضـایی زاده، ح.، چوپـانی، ر.، و انوشـیروانی، م. (1396). موروری بار کلیات تبه سوناتی ایرانی (مروری بر طب سـنتی ایران) (چاپ هجدهم). انتشارات طب سناتی ایران.

[18] Kurtik, G. E. (2021). On the origin of the 12 zodiac constellation system in ancient Mesopotamia. Journal for the History of Astronomy, 52(1), 53-66. DOI:10.1177/0021828620980544.

[19] Onsorol Maali Kikavos, A. Gh. Z. (1967). Qabus Namah (The king's life story) (p. 194). In M. Fallahzadeh, Persian writing on music: A study of Persian musical literature from 1000 to 1500 AD (Doctoral dissertation). Uppsala University.

[۲۰] اشمیت پیترز، ج. (۲۰۱۰). موسیقی‌درمانی: درآمدی بر آن، به همراه پیوست: موسیقی‌درمانی در تاریخ ادبیات ایران، نوشته‌ی ع. زاده‌محمدی (ویرایش چهارم). انتشارات اسرار دانش.

[21] شیرازی، ف. د. (1996). بحرالاحان (چاپ دوم). انتشارات فروغی. [فارسی]. 1، 16.

[22] تقی‌بینش، م. (1997). شـناخت موسـیقی ایرانـی (چـاپ اول). انتشارات دانشگاه هنر.

[23] Villoteau, J. P. G. R. (1807). Mémoire sur la musique. Dans Description de l'Égypte (vol. XIII et XIV).

[۲۴] زاده‌محمدی، ع. (۱۳۷۹). چشم‌اندازهای نـو در موسـیقی ایـران (ویرایش اول). انتشارات سرود.

[25] Zarka, P. (2009). Astronomy and astrology. Proceedings of the International Astronomical Union, 5(S260), 420- 425. DOI:https://doi.org/10.1017/S1743921311002602 1

[26] *Masih Aslani*,. A systematic and innovative model for identifying the psychological effects and states induced by musical rhythm and tempo, based on the four temperaments (the four humors). International Journal of *Science, Culture and Art* 2 (1) (1405) 48-55.

[27]اصلانی، مسیح، (1405) ،یک مدل عددی نوآورانه برای شناسایی تأثیرات و حالات روانیِ ریتم و تمپوی موسیقی بر اساس چهار مزاج (طبع) ، نشریه علم، فرهنگ و هنر، 2(1)، 51-60.

[28] *Masih Aslani*,. Novel Numerical Formulas for Asymmetric Steps (17- and 24-Note Systems) to Induce Psychological Effects in Music Therapy. International Journal of *Science, Culture and Art* 2 (1) (1405) 16-28.

[29] *Masih Aslani*,. An innovative 17-note numerical system and its computational method for identifying the psychological effects and states of musical pitch. International Journal of ***Science, Culture and Art*** 1 (1) (2026), 1-15.

[30] اصلانی، مسیح، (1405) ، سیستم عددي نوآورانه 17 نتی و روش محاسباتی آن براي شناسایی اثرات و حالات روانشناختی گام موسیقی ، نشریه علم، فرهنگ و هنر، 1(2)، (1-16).

[31] اصلانی، مسیح، (1405) ، فرمول‌های عددی نوین برای گام‌های نامتقارن (سیستم‌های 17 و 24 نتی) جهت ایجاد اثرات روان‌شناختی در موسیقی‌درمانی ، نشریه علم، فرهنگ و هنر، 2(1)، 35-50.

[32]اصلانی، مسیح، (1405) ، سیستم عددی نوآورانه 24 نتی: روش محاسباتی و مدل رابطه بین نت‌ها، عناصر و سیارات ، نشریه علم، فرهنگ و هنر، 2(1)، 17-34.

[33] *Daniel Bikas*, A Critical Review of Iranian Conceptual Carpet Art. Iranian International Journal of ***Science, Culture and Art*** 1 (1) (1405) 58-71.

[34] *Fatemeh Etemadi Rad*,. Examining methods to defend workers' rights. International Journal of *Science, Culture and Art*, 2 (1) (1405) 54-57.

[35] اعتمادي راد، فاطمه، (1405) ، راهکارهای آموزش و پرورش و قهرمانی در جامعه، نشریه علم، فرهنگ و هنر، 1(2)، (61-64).

[36] اعتمادي راد، فاطمه، (1405) ، صنعت توریسم و حمایت از کالا، تولید و منافع ملی ، نشریه علم، فرهنگ و هنر،1(2)، (65- 68).

[37]پورشیخی، محسن، (1400). الگوریتمی جدید برای حل مساله فروشنده دوره گرد متقارن ، *نشریه علمی پژوهشی علم، فرهنگ و هنر*، *اول*(اول)، 8-18.

[38] *Mohsen Poursheikhi*. A new algorithm to solve the symmetric iterative Marketer problem. International Journal of *Science, Culture and Art* 1 (11) (2021) 12-18.

[39]شهریار ، بتول، (1399). اثر تنش سرمازدگی بر رشد گیاهان پاییزه و راهکارهاي مقابله ، نشریه علمی پژوهشی علم، فرهنگ و هنر ، 1 ،(1)، 7-9..

[40] *Batul Shahriyar, Ciruc Mansourifar*,The effect of frost stress on the growth of autumn plants and ways to

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deal with them. *Science, Culture and Art* 1 (1) (2021) 7-11.
[41] Reza Kakulvand,. Population balancing methods to describe the increase and decrease of particle size. Iranian International Journal of ***Science, Culture and Art,*** 1 (1) (1400) 19-25.
[42] *Reza Kakulvand*,. Population balancing methods to describe the increase and decrease of particle size. ***Science, Culture and Art,*** 1 (1) (1400) 19-25.
[43] کاکولوند ، رضا، (1400) . بررسی ضریب پسا در سیستم هاي گاز مایع، نشریه علمی پژوهشی علم، فرهنگ و هنر، اول (اول)، 1-7.
[44] *Reza Kakulvand*,. Population balancing methods to describe the increase and decrease of particle size. International Journal of ***Science, Culture and Art*** 1 (1) (1400) 19-25.
[45] کاکولوند، رضا، (1401). روش های موازنه ی جمعیت برای توصیف بزرگ شدن و کوچک شدن اندازه ذرات ، *نشریه علمی پژوهشی علم، فرهنگ و هنر، اول*(اول)، 19-23..
[46] *R. kakulvand*, Review of drag coefficients on gas – liquid tower: the drag coefficient independent and dependent on bubble diameter in bubble column experiment, *Chem. Rev. Lett.*, 2 (2019) 48-58.
[47] *R. Kakulvand*. Investigation of drag coefficient in gas-liquid systems. Journal of *Science Culture and Art*, 1 (1) (2021) 1 – 6.
[48] *R. Kakulvand*, Effect of Non-uniform Convection on Entropy Generation and Enthalpy for the Laminar Developing Pipe Flow of a High Prandtl Number Fluid. *World Journal of Applied Chemistry*, 4 (2019) 69-78.
[49] *R. kakulvand*, The effects of transient heat flux on the tube in contact with the natural convection, on enthalpy and entropy generation, for developed laminar flow of fluid with high Prandtl number. *Chem. Rev. Lett.*, 2 (2019) 165-175.
[50] *R. kakulvand,* The effects of transient radiant flow on pipe in contact with natural convection, for developed laminar flow of fluid with high Prandtl number, on enthalpy and entropy generation, Journal of *Chem. Rev. Lett.*, 2 (2019) 130-137.
[51] *R. kakulvand*, Effect of non-uniform temperature distribution on entropy generation and enthalpy for the laminar developing pipe flow of a high Prandtl number fluid. *Chem. Rev. Lett.*, 2 (2019) 98-106.

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